APPLICATION OF SCHWINGER'S ACTION PRINCIPLE TO QUANTISE A FOURTH ORDER MESON FIELD

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ABSTRACT. We have here applied Schwinger's Action Principle to the case of a fourth order meson equation proposed by Bhabha and Thirring. The result obtained thus is not new, but the method illustrates with the simplest model the difficulties of applying Action Principle when the Lagrangian contains even the second order derivatives of the field operator, and gives a concrete and complete example of the generalisation of the Action Principle when the Lagrangian contains higher order derivatives as given by the author in a recent paper.

INTRODUCTION

The fourth order meson equation proposed by Bhabha (1950) and Thirring (1950) has been considered in some detail, including field quantisation, by Thirring (1950). Here, however, we shall employ the general Action Principle of Schwinger (1951, 1953) in quantising this field, and obtaining a symmetric energy-momentum tensor of the same The result obtained thus is not new, but the method illustrates with the simplest model the difficulties of applying Action Principle when the Lagrangian contains even the second order derivatives of the field operator, and gives a concrete and complete example of the generalisation of the Action Principle when the Largrangian contains higher order derivatives as given by the author in a recent paper (Misra, 1959 ; hereafter to be referred as P1).

The above meson equation has been found useful in explaining anomalous magnetic moments of nucleons (Misra and Deo (1956) and is interesting since it gives rise to convergent contributions in many physical processes which involve meson propagator. The fourth order calculations of the matrix elements for the nuclear forces with this fourth order meson equation also gives rise to many conclusions of theoretical interest (Misra ..., to be published).

The free field fourth order meson equation is

$$(\Box - \kappa^2)^2 \phi(x) = 0 \qquad \dots \quad (1)$$

where

$$\Box \equiv (\vec{\nabla}^2 - (\partial_0)^2) \equiv g_{\mu\nu}\partial_\mu\partial_\nu .$$

Equation (1) can be obtained by using the invariant Lagrangian density

$$L(x) = -\frac{1}{2\kappa^2}(\Box\phi)(\Box\phi) - (\partial_\mu\phi)(\partial_\mu\phi) - \frac{k^2}{2}\phi^2 \qquad \dots (2)$$

which gives us the Action Integral W_{12} for any two space-like surfaces σ_1 and σ_2 as

$$W_{12} = \int_{\sigma_2}^{\sigma_1} L(x)d^4x.$$

This gives us, when the field equation (1) is satisfied,

$$\delta W_{12} = F(\sigma_1) - F(\sigma_2) \qquad \dots (3)$$

where, by equation (15) of PI,

$$F(\sigma) = \int_\sigma [\pi_\mu\delta_0\phi + \pi_{\mu\nu}\partial_\nu(\delta_0\phi) + L\delta x_\mu]\, d\sigma_\mu. \qquad \dots (4)$$

In the above, by equation (13) of PI,

$$\pi_\mu = \frac{\partial L}{\partial(\partial_\mu\phi)} - \partial_\nu \frac{\partial L}{\partial(\partial_\mu\partial_\nu\phi)}$$

$$= -2\partial_\mu\phi + \frac{1}{\kappa^2}\partial_\mu\Box\phi \qquad \dots (5)$$

and

$$\pi_{\mu\nu} = \frac{\partial L}{\partial(\partial_\mu\partial_\nu\phi)}$$

$$= -\frac{1}{\kappa^2} g_{\mu\nu}\Box\phi \qquad \dots (6)$$

COMMUTATION RELATION

Let us now substitute

$$\pi_{(0)} \equiv n_\mu\pi_\mu = -2\partial^{(0)}\phi + 1/\kappa^2\partial^{(0)}\Box\phi \qquad \dots (7)$$

and

$$\pi_{(00)} \equiv n_\mu n_\nu\pi_{\mu\nu} = 1/\kappa^2\Box\phi. \qquad \dots (7')$$

Then, by equation (23) of PI we obtain,

$$n_\mu\pi_\mu{}^{[\sigma,0]} = n_\mu(\pi_\mu - \partial_{t\nu}\pi_{\mu\nu})$$

$$= n_\mu\pi_\mu = \pi_{(0)} \qquad \dots (8)$$

and

$$n_\mu \pi_\mu^{[\sigma,1]} = n_\mu(-n_\nu \pi_{\mu\nu})$$
$$= -\pi_{(00)} \qquad \ldots \quad (9)$$

Hence equation (27) of PI gives us

$$[\phi(x), \int_\sigma \{\pi_{(0)}(x')\ \partial_0\phi(x') - \pi_{(00)}(x')\delta_0((\partial'^{(0)})\phi(x'))\}]d\sigma(x') = i\delta_0\phi(x) \qquad \ldots \quad (10)$$

and

$$[\partial^{(0)}\phi(x), \int_\sigma \{\pi_{(0)}(x')\delta_0\phi(x') - \pi_{(00)}(x')\delta_0((\partial'^{(0)})\phi(x'))\}]d\sigma(x') = i\delta_0(\partial^{(0)}\phi(x) \ldots \quad (11)$$

Hence, using equation (31) of PI, we obtain, with the notation $\delta_\sigma(x-x') = n_\mu \delta_\mu^{(\sigma)}(x-x')$,

$$[\phi(x),\ \pi_{(0)}(x')\delta_0\phi(x')] = i\delta_\sigma(x-x')\delta_0\phi(x),$$
$$[\phi\partial^{(0)}\phi(x), \pi_{(00)}(x')\delta_0(\partial'^{(0)}\phi(x'))] = -i\delta_\sigma(x-x')\delta_0(\partial^{(0)}\phi(x)), \qquad \ldots \quad (12)$$

and

$$[\phi(x),\ \pi_{(00)}(x')\ \delta_0(\partial'^{(0)}\phi(x'))]$$
$$= [\partial^{(0)}\phi(x),\ \pi_{(0)}(x')\delta_0\phi(x')] = 0. \qquad \ldots \quad (12')$$

When we take

$$[\phi(x),\ \delta_0\phi(x')] = [\phi(x),\ \delta_0(\partial'^{(0)}\phi(x'))]$$
$$= [\partial^{(0)}\phi(x),\ \delta\ \phi(x')]$$
$$= [\partial^{(0)}\phi(x),\ \delta_0(\partial'^{(0)}\phi(x'))] = 0, \qquad \ldots \quad (13)$$

which have to be satisfied if we assume that the variations of the field operators allowed are such that the commutators or their derivatives do not change, then we obtain the commutation relationships for space-like separation of the points x and x' as

$$[\phi(x),\ \pi_{(00)}(x')] = [\partial^{(0)}\phi(x),\ \pi_{(0)}(x')] = 0 \qquad \ldots \quad (14)$$

and

$$[\phi(x),\ \pi_{(0)}(x')] = i\delta_\sigma\ (x-x'),$$
$$[\partial^{(0)}\phi(x),\ \pi_{(00)}(x')] = -i\delta_\sigma(x-x'). \qquad \ldots \quad (15)$$

Hence making use of equations (7) we obtain the commutation relations for space-like separation of the points

$$[\phi(x),\ -2\partial'^{(0)}\phi(x') + \frac{1}{\kappa^2}\partial'^{(0)}\Box'\phi(x')] = i\delta_\sigma(x-x'),$$

$$[\phi(x),\ -\Box'\ \phi(x')] = 0,$$

$$[\partial^{(0)}\phi(x),\ -2\partial'^{(0)}\phi(x') + \frac{1}{\kappa^2}\ \partial'^{(0)}\Box'\ \phi(x')] = 0$$

and

$$[\partial^{(0)}\phi(x), -\frac{1}{\kappa^2}\Box'\phi(x')] = i\delta_0(x-x'). \qquad \ldots (16)$$

Let us now put, for *any* two space-time points x and y,

$$[\phi(x), \phi(y)] = C(x-y), \qquad \ldots (17)$$

where the right hand side is a function of the differences of the coordinates since it has to be invariant under translations. Using this, we obtain that in equations (16), while operating on the commutator $C(x-x')$, $\partial'^{(0)} = -\partial^{(0)}$ and $\Box' = \Box$. Also, since n_μ is sufficiently stationary at any point of the surface, $\partial'^{(0)}\Box' = \Box'\partial'^{(0)}$. Hence, by straight-forward simplification, equations (16) with x and x' having space-like separation, reduce to

$$\partial^{(0)}\Box\, C(x-x') = -i\kappa^2\delta_0(x-x'),$$

$$\partial^{(0)}C(x-x') = \Box\, C(x-x') = 0. \qquad \ldots (18)$$

and

$$2(\partial^{(0)})^2C(x-x') - \frac{1}{\kappa^2}(\partial^{(0)})^2\Box\, C(x-x') = 0. \qquad \ldots (19)$$

Our equations now are sufficiently simple for us to obtain the value of $C(x-y)$ for arbitrary separation of the points x and y, which we had been unable to do in the general case of PI. For this purpose, we shall use the solution of the field operator in terms of the advanced and retarded Green's functions of the field equation (1) (Jauch and Rohrlich (1955)). The equations that these Green's functions satisfy are

$$(\Box - k^2)^2G_{R,A}(x-x') = -\delta_4(x-x'), \qquad \ldots (20)$$

where

$$G_R(x-x') = 0 \qquad \text{when} \quad x'_0 > x_0,$$

and

$$G_A(x-x') = 0 \qquad \text{when} \quad x'_0 < x_0. \qquad \ldots (21)$$

Replacing x by x' in the field equation (1) and using equation (20) we obtain,

$$\phi(x) = \int_{\sigma_2}^{\sigma_1} [G_{R,A}(x-x')(\Box'-\kappa^2)^2\phi(x')-(\Box-k^2)^2G_{R,A}(x-x')\phi(x')]d^4x'$$

$$= \int_{\sigma_2}^{\sigma_1} [G_{R,A}(x-x')(\Box'^2-2\kappa^2\Box')\phi(x')-(\Box^2-2\kappa^2\Box)G_{R,A}(x-x')\phi(x')]d^4x' \qquad \ldots (22)$$

where σ_1 and σ_2 are two space-like surfaces with σ_1 later than x and σ_2 earlier than x in time, such that the point x lies between them. To simplify the right hand side of equation (22), we first note that

$$G_{R,A}(x-x')\Box'^2\phi(x') - \Box^2 G_{R,A}(x-x')\phi(x')$$
$$= \partial'_\mu[G_{R,A}(x-x')\partial'_\mu\Box'\phi(x') - \partial'_\mu G_{R,A}(x-x')\Box'\phi(x')$$
$$+\Box' G_{R,A}(x-x')\partial'_\mu\phi(x') - \partial'_\mu\Box' G_{R,A}(x-x')\phi(x')] \quad \ldots \quad (23)$$

and

$$G_{R,A}(x-x')\Box'\phi(x') - \Box G_{R,A}(x-x')\phi(x')$$
$$= \partial'_\mu[G_{R,A}(x-x')\partial'_\mu\phi(x') - \partial'_\mu G_{R,A}(x-x')\phi(x')]. \quad \ldots \quad (24)$$

We apply Gauss theorem to the right hand side of equation (22), and use the invariance of Green's functions and the equations (21). These Green's functions vanish for space-like separation of the points x and x', and thus the integral j over the surface joining σ_1 and σ_2 for infinite space-like separation also vanishes. Hence we get,

$$\phi(x) = \int_{\sigma_1}[G_{R,A}(x-x')\partial_\mu'\Box'\phi(x') - \partial'_\mu G_{R,A}(x-x')\Box'\phi(x')$$
$$+\Box' G_{R,A}(x-x')\partial'_\mu\phi(x') - \partial'_\mu\Box' G_{R,A}(x-x')\phi(x')$$
$$-2\kappa^2 G_{R,A}(x-x')\partial'_\mu\phi(x') + 2\kappa^2\partial'_\mu G_{R,A}(x-x')\phi(x')]d\sigma_\mu(x')$$
$$-\int_{\sigma_2}(\text{the same integrand})\ d\sigma_\mu(x'). \quad \ldots \quad (25)$$

In solution (25) of $\phi(x)$ in terms of the surface integrals, on using equation (21) for advanced and retarded Green's functions, we obtain,

$$\phi(x) = \int_{\sigma_1}[G_A(x-x')\partial_\mu'\Box\phi'(x') - \partial_\mu' G_A(x-x')\Box'\phi(x')$$
$$+\Box' G_A(x-x')\partial_\mu'\phi(x') - \partial_\mu'\Box' G_A(x-x')\phi(x')$$
$$-2\kappa^2 G_A(x-x')\partial_\mu'\phi(x') + 2\kappa^2\partial_\mu' G_A(x-x')\phi(x')]d\sigma_\mu(x') \quad \ldots \quad (26)$$

$$= -\int_{\sigma_2}(\text{the same integrand with } G_A \text{ replaced by } G_R)\ d\sigma_\mu(x') \quad \ldots \quad (27)$$

We now substitute

$$G(x-x') = G_R(x-x') - G_A(x-x'), \quad \ldots \quad (28)$$

such that, on the surface σ_1, $G=-G_A$ and on the surface σ_2, $G=G_R$. Hence we obtain for any surface σ which does not contain x

$$\phi(x) = -\int_\sigma [G(x-x')\partial_\mu{}'\Box'\phi(x') - \partial_\mu{}'G(x-x')\Box'\phi(x')$$
$$+\Box'G(x-x')\partial_\mu{}'\phi(x') - \partial_\mu{}'\Box'G(x-x')\phi(x')$$
$$-2\kappa^2 G(x-x')\partial_\mu{}'\phi(x') + 2\kappa^2\partial_\mu{}'G(x-x')\phi(x')]d\sigma_\mu(x') \qquad \ldots \quad (29)$$

Let us now assume that the above space-like surface σ contains the point y. Hence we get, by equation (29),

$$[\phi(x), \phi(y)] = \int_\sigma \{-G(x-x')\partial_\mu{}'\Box'[\phi(x'), \phi(y)]$$
$$+\partial_\mu{}'G(x-x')\Box'[\phi(x'), \phi(y)] - \Box'G(x-x')\partial_\mu{}'[\phi(x'), \phi(y)]$$
$$+\partial_\mu{}'\Box'G(x-x')[\phi(x'), \phi(y)] + 2\kappa^2 G(x-x')\partial_\mu{}'[\phi(x'), \phi(y)]$$
$$-2\kappa^2\partial_\mu{}'G(x-x')[\phi(x'), \phi(y)]\}n_\mu(x')d\sigma(x'),$$

which on rearrangement becomes, by definition (17),

$$C(x-y)$$
$$= \int_\sigma \{G(x-x')(2\kappa^2\partial'^{(0)}C(x'-y) - \partial'^{(0)}\Box'C(x'-y))$$
$$-\partial'^{(0)}G(x-x')(-\Box'C(x'-y) - 2\kappa^2 C(x'-y))$$
$$-\Box'G(x-x')\partial^{(0)}C(x'-y) + \partial'^{(0)}\Box'G(x-x')C(x'-y)\}d\sigma(x').$$

Using equations (18), the above relationship reduces to

$$C(x-y) = \int_\sigma G(x-x')\{i\kappa^2\delta_0(x'-y)\}d\sigma(x')$$

such that

$$C(x-y) = i\kappa^2 G(x-y) \qquad \ldots \quad (30)$$

We shall now write down explicitly the retarded and advanced Green's functions as contour integrals. We have,

$$G_R(x) = \frac{-1}{(2\pi)^4}\int_\frown \frac{e^{i(\vec{k}\cdot\vec{x} - k_0 x_0)}}{(k^2+\kappa^2)^2}\, d^3k\, dk_0 \qquad \ldots \quad (31)$$

where the contour C_R is defined by figure 1a in the complex k_0 plane. This can be easily seen when we notice that

$$\int \frac{e^{-ik_0 x_0}}{(k^2+\kappa^2)}\, dk_0$$

over an infinite semi-circle in the upper half plane vanishes when $x_0 < 0$. Hence by Cauchy's theorem the right hand side of equation (31) vanishes when $x_0 < 0$, which corresponds to the first of equations (21). We can write in a similar way, with C_A defined by figure 1a,

$$G_A(x) = \frac{-1}{(2\pi)^4} \int_{C_A} \frac{e^{i(\vec{k}\cdot\vec{x}-k_0 x_0)}}{(k^2+\kappa^2)^2}\, d^3k\, dk_0 \qquad \dots (32)$$

which can be shown to satisfy equation (21) by considering the vanishing of the above integral over an infinite semi-circle in the lower half complex k_0 plane.

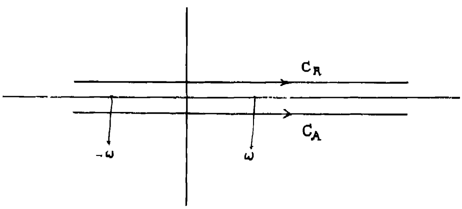

Fig 1(a)

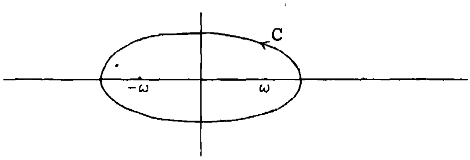

Fig. 1(b)

Thus we have,

$$G(x) = \frac{1}{(2\pi)^4} \int_C \frac{e^{i(\vec{k}\cdot\vec{x}-k_0 x_0)}}{(k^2+\kappa^2)^2}\, d^3k\, dk_0, \qquad \dots (33)$$

where C is the contour of figure 1b. Integrating for k_0 this result gives us,

$$G(x) = \frac{1}{2(2\pi)^3} \int \frac{d^3k}{\omega^3}\, e^{i\vec{k}\cdot\vec{x}} [(\omega x_0)\cos(\omega x_0) - \sin(\omega x_0)] \qquad \dots (34)$$

with $\qquad \omega = +\sqrt{\vec{k}^2+\kappa^2}\,.$

We can now verify that $C(x-x') = i\kappa^2\, G(x-x')$ satisfies equations (18) and (19) for space-like separation of the points x and x'. This is directly demonstrated when we evaluate the left hand side of the covariant equations (18) and (19), substituting the value of $C(x-x')$ according to equations (30) and (34), and then proving the validity of equations (18) and (19) with the special coordinate system $x_0 - x'_0 = 0$. Equation (19), which was not used to deduce equation (30), may be specially seen to be satisfied.

ENERGY-MOMENTUM TENSOR

We can also obtain the energy-momentum tensor for this particular case from the generator $F(\sigma)$ on the surface σ. For this purpose we first recollect the equation

$$F(\sigma) = \int_\sigma [\pi_\mu \delta_0\phi + \pi_{\mu\nu}\delta_0(\partial_\nu\phi) + L\delta x_\mu] d\sigma_\mu$$

Now, let $\delta\phi$ be the total variation of ϕ due to the intrinsic variation $\delta_0\phi$ on the surface σ and also due to a *rigid* displacement of the surface by δx_μ. Then we have,

$$\delta\phi = \delta_0\phi + \phi(x+\delta x) - \phi(x) = \delta_0\phi + (\partial_\kappa\phi)\delta x_\kappa \qquad \dots \quad (35)$$

such that

$$\partial_\nu(\delta\phi) = \partial_\nu(\delta_0\phi) + (\partial_\nu\partial_\kappa\phi)\delta x_\kappa + (\partial_\kappa\phi)(\partial_\nu(\delta x_\kappa)). \qquad \dots \quad (36)$$

Hence, using the above two equations we obtain,

$$F(\sigma) = \int_\sigma [\pi_\mu\delta\phi + \pi_{\mu\nu}\partial_\nu(\delta\phi) + L\delta x_\mu - \pi_\mu(\partial_\nu\phi)\delta x_\nu - \pi_{\mu\nu}(\partial_\kappa\partial_\nu\phi)\delta x_\kappa$$
$$- \pi_{\mu\nu}(\partial_\kappa\phi)\partial_\nu(\delta x_\kappa)] d\sigma_\mu \qquad \dots \quad (37)$$

We now note that for a rigid displacement of the surface, since two neighbouring points on the original surface and on the displaced surface will remain at the same distance, we shall have,

$$\partial_\kappa(\delta x_\mu) + \partial_\mu(\delta x_\kappa) = 0 \qquad \dots \quad (38)$$

But the above is the condition that the transformation $x_\mu \to x'_\mu = x_\mu + \delta x_\mu$ is an infinitesimal Lorentz transformation. This gives us the equivalence of the rigid displacement to a Lorentz transformation.

Let us now consider an infinitesimal Lorentz transformation (or a rigid displacement) $x_\mu \to x_\mu + \delta x_\mu$, and take the intrinsic variation $\delta_0\phi$ such that the total variations $\delta\phi$ and $\delta(\partial^{(0)}\phi)$ vanish. Then, for such intrinsic variations, the field operators will remain the same on the surfaces σ and $\sigma + \delta\sigma$. The expression for the generator on the surface σ now becomes

$$F_{\delta x}(\sigma) = \int_\sigma [L\delta x_\mu - \pi_\mu(\partial_\nu\phi)\delta x_\nu - \pi_{\mu\kappa}(\partial_\nu\partial_\kappa\phi)\delta x_\nu - \pi_{\mu\kappa}(\partial_\nu\phi)\partial_\kappa(\delta x_\nu)] d\sigma_\mu \qquad \dots \quad (39)$$

The above formula does not contain any spin-dependent term since we have taken a scalar field and thus the formula (35) did not involve any expression for the change due to a rotation in the coordinate space that the δx_μ contains.

The expression (39) on the right hand side depends only on the transformation or displacement given by δx_μ It also involves an integration over σ of functions of field operator However, since the total variations $\delta\phi$ and $\delta(\partial^{(0)}\phi)$ are zero, these field operators do not change for an infinitesimal displacement of the surface and hence remain tho samo for a succession of such displacements of the surface. A succession of such displacements is equivalent to a number of infinitesimal Lorentz transformations, under which our theory is invariant. Hence the generator $F(\sigma)$ given by equation (39) is the samo for all surfaces, and depends only on δx_μ.

In order to obtain a symmetrical energy-momentum tensor, we need at this stage some further manipulations similar to the treatment of the term due to spin by Schwinger (1951). Equation (38) gives us

$$\pi_{\mu\kappa}(\partial_\nu\phi)\partial_\kappa(\delta x_\nu) = \tfrac{1}{2}\chi_{\mu\kappa\nu}\partial_\kappa(\delta x_\nu)$$

where

$$\chi_{\mu\kappa\nu} = [\pi_{\mu\kappa}(\partial_\nu\phi) - \pi_{\mu\nu}(\partial_\kappa\phi)] \qquad \dots \quad (40)$$

We now define

$$2f_{\mu\kappa\nu} = \chi_{\kappa\mu\nu} - \chi_{\kappa\nu\nu} + \chi_{\nu\kappa\mu} = \chi_{\mu\kappa\nu} + \chi_{\kappa\nu\mu} + \chi_{\nu\kappa\mu} \qquad \dots \quad (41)$$

Clearly,

$$f_{\kappa\mu\nu} = -f_{\mu\kappa\nu}, \qquad \dots \quad (42)$$

and

$$\tfrac{1}{2}\chi_{\mu\kappa\nu}\partial_\kappa(\delta x_\nu) = f_{\mu\kappa\nu}\partial_\kappa(\delta x_\nu). \qquad \dots \quad (43)$$

Using equations (6) and (7′), we now obtain, by equations (40) and (41) and on simplification,

$$f_{\mu\kappa\nu} = [\pi_{\kappa\nu}(\partial_\mu\phi) - \pi_{\mu\nu}(\partial_\kappa\phi)]. \qquad \dots \quad (44)$$

Consistent with earlier notations, let tho suffix (0) denote the normal component of any tensor at any point of the surface and the suffix $(t\mu)$ denote the tangential component of any tensor at any point of tho surface. Then we have,—

$$\int_\sigma \partial_\kappa(f_{\mu\kappa\nu}\delta x_\nu)d\sigma_\mu = \int_\sigma \partial_\kappa(f_{(0)\kappa\nu}\delta x_\nu)d\sigma$$

$$= \int \partial_{(t\kappa)}(f_{(0)(t\kappa)\nu}\delta x_\nu)d\sigma + \int \partial_{(0)}(f_{(0)(0)\nu}\delta x_\nu)d\sigma = 0$$

provided that $f_{(0)(\iota k)\nu}\delta x_\nu$ vanishes sufficiently rapidly at the boundary of the surface σ, which we assume to be the case. Hence in equation (39) we obtain,

$$\int_\sigma \pi_{\mu\kappa}(\partial_\nu\phi)\partial_\kappa(\delta x_\nu)d\sigma_\mu = \int_\sigma f_{\mu\kappa\nu}\partial_\kappa(\delta x_\nu)d\sigma_\mu$$

$$= - \int_\sigma \partial_\kappa(f_{\mu\kappa\nu})\delta x_\nu d\sigma_\mu = \int_\sigma \partial_\kappa(f_{\kappa\mu\nu})\delta x_\nu d\sigma_\mu$$

and thus we finally get,

$$- F_{\delta x}(\sigma)$$

$$= - \int_\sigma [g_{\mu\nu}L - \pi_\mu(\partial_\nu\phi) - \pi_{\mu\kappa}(\partial_\nu\partial_\kappa\phi) - \partial_\kappa(f_{\kappa\mu\nu})]\delta x_\nu d\sigma_\mu = \int T_{\mu\nu}\delta x_\nu d\sigma_\mu \qquad \ldots \quad (45)$$

where

$$T_{\mu\nu}$$

$$= \pi_\mu(\partial_\nu\phi) + \pi_{\mu\kappa}(\partial_\nu\partial_\kappa\phi) + \partial_\kappa(f_{k\mu\nu}) - Lg_{\mu\nu}. \qquad \ldots \quad (46)$$

This gives us, on substitution of the values in our particular case, by equations (7), (7′) and (44),

$$T_{\mu\nu} = g_{\mu\nu}\left[-\frac{1}{2\kappa^2}(\Box\phi)(\Box\phi) - \frac{1}{\kappa^2}(\partial_\lambda\Box\phi)(\partial_\lambda\phi) + (\partial_\lambda\phi)(\partial_\lambda\phi) + \frac{\kappa^2}{2}\phi^2\right]$$

$$+ \frac{1}{\kappa^2}[(\partial_\mu\Box\phi)(\partial_\nu\phi) + (\partial_\nu\Box\phi)(\partial_\mu\phi)] - 2(\partial_\mu\phi)(\partial_\nu\phi). \qquad \ldots \quad (47)$$

The above $T_{\mu\nu}$ is obviously symmetric in μ and ν, and we shall presently see that it represents the energy-momentum tensor. For this purpose, let

$$\delta x_\mu = \epsilon_\mu + \epsilon_{\mu\nu}x_\nu \qquad \ldots \quad (48)$$

be the infinitesimal Lorentz transformation associated with the rigid displacement. Then the momentum and angular momentum operators are defined by (Jauch and Rohrlich (1955) p.12)

$$U = 1 + iK = 1 + i(P_\nu\epsilon_\nu + \tfrac{1}{2}J_{\nu\lambda}\epsilon_{\nu\lambda})$$

where the state-vector transforms by the unitary operator U, and thus we have,

$$K = -F_{\delta x}(\sigma) = P_\nu(\sigma)\epsilon_\nu + \tfrac{1}{2}\, J_{\nu\lambda}(\sigma)\epsilon_{\nu\lambda}, \qquad \ldots \quad (49)$$

where the total energy-momentum four-vector is given as

$$P_\nu(\sigma) = \int_\sigma T_{\mu\nu}d\sigma_\mu \qquad \ldots \quad (50)$$

and the total angular momentum tensor is given as

$$J_{\nu\lambda}(\sigma) = \int_\sigma M_{\mu\nu\lambda}d\sigma_\mu \qquad \ldots \quad (51)$$

with

$$M_{\mu\nu\lambda} = (T_{\mu\nu}x_\lambda - T_{\mu\lambda}x_\nu). \qquad \ldots (52)$$

The conservation laws follow from the earlier remark that the generator $F_{\delta x}(\sigma)$ is independent of the surface σ, since a diplacement of the surface is equivalent to a Lorentz transformation, and our theory is Lorentz invariant.

Here we may note a difficulty of this theory The energy density of the field for a flat surface $t = x_0 =$ const. is given as

$$-T_{00} = (\vec{\nabla}\phi)^2 + (\partial_0\phi)^2 + \frac{\kappa^2}{2}\phi^2$$

$$-\frac{1}{2\kappa^2}(\Box\phi)(\Box\phi) - \frac{1}{\kappa^2}(\vec{\nabla}\Box\phi)\;(\vec{\nabla}\phi)\quad\frac{1}{\kappa^2}(\partial_0\Box\phi)(\partial_0\phi). \qquad \ldots (53)$$

The above expression is *not* a positive definite function of the field operators. This leads to the suspicion that this theory may lead to inconsistent results associated with "ghost" states However, such states are known to exist in the exact solutions of the semi-relativistic Lee model (Kallen and Pauli, 1955) and are also suspected to be present even in quantum electrodynamics (Landau, 1955). Since this defect seems to be rather a persistent feature of field theory, we consider it worthwhile to see the effect of this theory on nuclear forces when the mesons occur in the virtual state. This might be interesting since it has been pointed out by Ferreti (1958) that it is possible to eliminate negative energy states for real particles in case of Lee model.

In any case, this is an interesting application of Action Principle to the case when the Lagrangian contains second order derivatives of the field operator

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